CHAPTER 4

Motion in a Plane

Topic Covered

INTRODUCTION

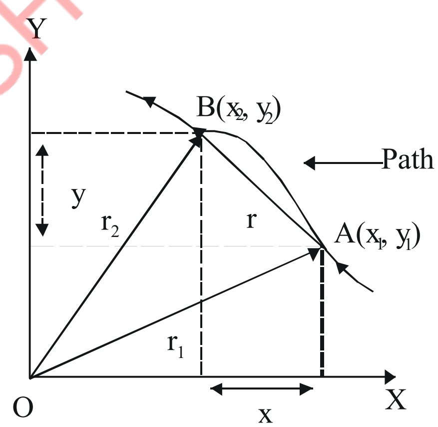
For a particle moving along a straight line, all the vector quantities:- position, velocity, displacement and acceleration have only one non-zero component and hence can be treated as positive or negative numbers.

But when a particle moves along a curve, each of these quantities can have two non-zero components. Hence the motion is said to be two dimensional because two components X and Y are associated with a vector quantity.

MOTION IN A PLANE

Displacement

To specify the position of an object the concept of the **position vector** needs to be introduced. The **position vector** is defined as a vector that starts at the origin and ends at the current position of the object (see Figure). In general, the position vector will be time dependent $\vec{r}(t)$. The position vector in terms of its components:



Displacement $\vec{r} = \vec{r}_2 - \vec{r}_1$, (According to Δ law of addition) $\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j}$

Velocity

The **velocity** of an object in two or three dimensions can be given as :

$$\vec{V} = \frac{\vec{dr}}{dt}$$

This equation shows that the velocity of an object in two or three dimensions is also a vector. Again, the velocity vector can be decomposed into its three components:

$\vec{V} = V_x(t)\hat{x} + V_y(t)\hat{y}$

The components of $\vec{v}(t)$ can be calculated from the corresponding components of the position vector $\vec{r}(t)$:

$$\vec{V}(t) = \frac{dx}{dt}\hat{x} + \frac{dy}{dt}\hat{y} = V_x(t)\hat{x} + V_y(t)\hat{y}$$

Acceleration

The **acceleration** of an object can be written as:

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{d^2r}{dt^2}$$

This equation shows that the acceleration of an object in two or three dimensions is also a vector, which can be decomposed into three components:

$$\vec{a}(t) = a_x(t)\hat{x} + a_y(t)\hat{y} + a_z + \hat{Z}$$

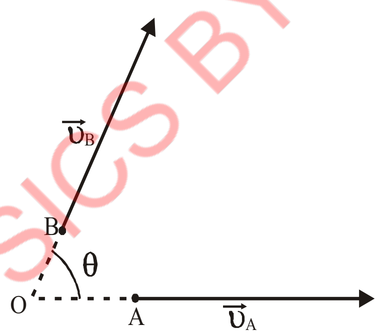
The components of (t) can be calculated from the corresponding components of the position vector (t) and the velocity vector (t):

$$\vec{a}(t) = a_x(t)\hat{x} + a_y(t)\hat{y} = \frac{d^2x}{dt^2}\hat{x} + \frac{d^2y}{dt^2}\hat{y}$$

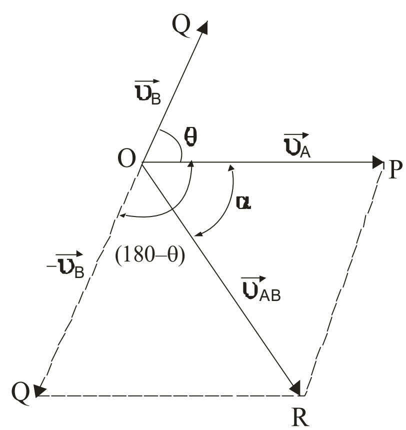
RELATIVE VELOCITY

In kinematics, **relative velocity** is the vector difference between the velocities of two objects, as evaluated in terms of a single coordinate system, usually an inertial frame of reference unless specifically stated otherwise.

Consider two bodies A and B moving with velocities $\vec{V}_A$ and $\vec{V}_B$ respectively and making an angle θ with each other as shown in figure



(a)



(b)

(i) The relative velocity of body A with respect to the body B is given by

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B = \vec{V}_A + \left(-\vec{V}_B\right) \qquad ...(1)$$

Velocity $\vec{V}_A$ is represented by $\overrightarrow{OP}$ and velocity $\vec{V}_B$ is represented by $\overrightarrow{OQ}$ as shown in figure (b). To find the magnitude of $\vec{V}_{AB}$, draw $\overrightarrow{OQ'} = -\vec{V}_B$ (i.e velocity equal and opposite to the velocity of body B). Now, the angle between $\vec{V}_A$ and $-\vec{V}_B$ is ($180° - \theta$) as shown in figure (b). $\vec{v}_{AB}$ is represented by the diagonal $\overrightarrow{OR}$ of the parallelogram OPRQ′.

Magnitude of $\vec{v}_{AB}$ is given by

$$\left|\vec{V}_{AB}\right| = \sqrt{v_A^2 + v_B^2 + 2v_A v_B \cos(180° - \theta)} \quad \text{(parallelogram law of vectors addition)}$$

Since $\quad \cos(180 - \theta) = -\cos\theta$

$$\therefore \quad \left|\vec{V}_{AB}\right| = \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos\theta} \quad \text{...(2)}$$

Direction of $\vec{V}_{AB}$

Let $\vec{V}_{AB}$ make an angle α with the direction of $\vec{V}_A$, then

$$\tan\alpha = \frac{V_B \sin(180° - \theta)}{V_A + V_B \cos(180° - \theta)} = \frac{V_B \sin\theta}{V_A - V_B \cos\theta} \quad (\because \sin(180° - \theta) = \sin\theta)$$

or
$$\alpha = \tan^{-1}\left(\frac{V_B \sin\theta}{V_A - V_B \cos\theta}\right) \quad \text{...(3)}$$

(ii) The relative velocity of body B with respect to the body A is given by

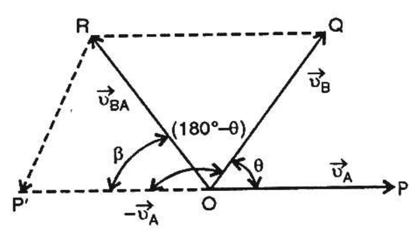
$$\vec{V}_{AB} = \vec{V}_B - \vec{V}_A = \vec{V}_B + \left(-\vec{V}_A\right) \quad \text{...(4)}$$

To find the magnitude of $\vec{V}_{BA}$, draw $\overrightarrow{OP'} = -\vec{V}_A$ (i.e. velocity equal and opposite to the velocity of body A). Complete the parallelogram OP′RQ as shown in figure (c). The magnitude of $\vec{v}_{BA}$ is given by the diagonal OR of the parallelogram OP′RQ

Using parallelogram law of vectors addition, we get

$$\left|\vec{V}_{BA}\right| = \sqrt{V_A^2 + V_B^2 + 2V_A V_B \cos(180 - \theta)}$$

or
$$\left|\vec{V}_{BA}\right| = \sqrt{V_A^2 + V_B^2 - 2V_A V_B \cos\theta} \quad \text{...(5)}$$



Direction of $\vec{V}_{BA}$

Let $\vec{V}_{BA}$ make an angle β with $-\vec{V}_A$, then

$$\tan\beta = \frac{V_B \sin(180° - \theta)}{V_A + V_B \cos(180° - \theta)} = \frac{V_B \sin\theta}{V_A - V_B \cos\theta}$$

or
$$\beta = \tan^{-1}\left(\frac{V_B \sin\theta}{V_A - V_B \cos\theta}\right) \quad \text{...(6)}$$

Thus, we find that the values of $\left|\vec{V}_{AB}\right|$ and $\left|\vec{V}_{BA}\right|$ are the same but their directions are different.

SPECIAL CASE

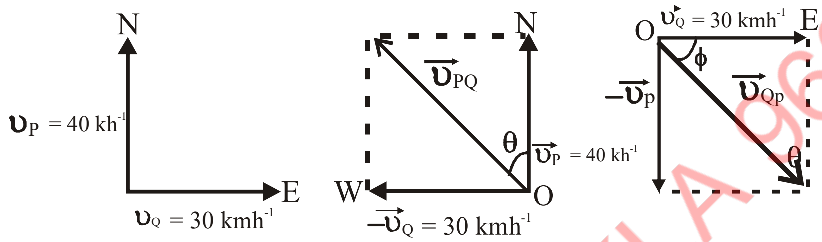
If both the bodies are moving at right angle to each other i.e. $\theta = 90^o$.

Hence from eqns. (2) and (5), we get

$$\left|\vec{V}_{AB}\right| = \left|\vec{V}_{BA}\right| = \sqrt{v_A{}^2 + v_B{}^2}$$

EXAMPLE

A bus P is travelling with a speed of 40 km/h towards N. Another bus Q is travelling with a speed of 30 km/h towards E. Find the magnitudes and direction of teh velocity of bus P w.r.t the bus Q and velocity of the bus Q w.r.t bus P.



The given problem is shown in figure

(i) Relative velocity of bus P w.r.t. Q is given by

$\vec{v}_{PQ} = \vec{v}_P - \vec{v}_Q$

Magnitude of relative velocity of bus P w.r.t. the bus Q is given by

$$v_{PQ} = \sqrt{v_P{}^2 + v_Q{}^2} = \sqrt{(40)^2 + (30)^2} = \sqrt{2500} = 50\ \text{kmh}^{-1}$$

Direction of v_{PQ} is given by

$$\tan\theta = \frac{v_Q}{v_P} = \frac{30}{40} = 0.7500$$

or $\theta = \tan^{-1}(0.7500) = 36^o 52'$

Thus, relative velocity of the bus P makes an angle of $36^o 52'$ with the North of West.

(ii) Relative velocity of bus Q w.r.t. P is given by

$\vec{v}_{QP} = \vec{v}_Q - \vec{v}_P$

Magnitude of the relative velocity of bus Q w.r.t. the bus P

$$\left|\vec{v}_{QP}\right| = \sqrt{v_P{}^2 + v_Q{}^2} = \sqrt{(40)^2 + (30)^2} = \sqrt{2500} = 50\,\text{kmh}^{-1}$$

Direction of v_{QP} is given by

$$\tan\phi = \frac{v_P}{v_Q} = \frac{40}{30} = 1.3333$$

or $\phi = \tan^{-1}(1.3333) = 53^o 8'$

Thus, relative velocity of the bus Q makes an angle of $53^o 8'$ with the East of South.

Some cases of Relative Motions observed in Daily Life

(a) Rain Fall

Let us suppose, rain falls vertically downward with a velocity $\vec{v_r}$ and the man is moving horizontally with a velocity $\vec{v}_m$ as shown in figure. The relative velocity of rain w.r.t. man is given by

$$\vec{v}_{rm} = \vec{v}_r + (-\vec{v}_m) = \vec{v}_r - \vec{v}_m$$

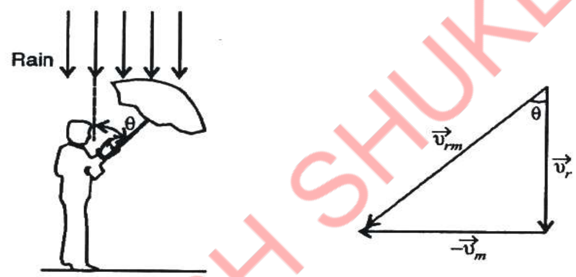
as shown in figure

Magnitude of $v_{\vec{rm}}$ is given by

$$\left|\vec{v}_{rm}\right| = \sqrt{v_r^{\,2} + v_m^{\,2}}$$

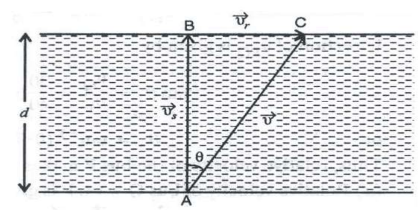
The direction of $v_{\vec{rm}}$ with the vertical is given by $\tan\theta = \frac{v_m}{v_r}$ or $\theta = \tan^{-1}\left(\frac{v_m}{v}\right)$

Thus the man has to hold his umbrella at an angle $\theta = \tan^{-1}\left(\frac{v_m}{v_r}\right)$ with the vertical to save himself from wetting as shown in figure



(b) River Flow

1. Let us suppose water is flowing with velocity $\vec{v}_r$ towards the east and a swimmer jumps into the river from one bank of the river in a direction perpendicular to the direction of the flow as shown in figure. Let velocity of swimmer be $\vec{v}_s$.



Then, the net speed of the swimmer is given by

$$\left|\vec{v}\right| = \sqrt{v_r^{\,2} + v_s^{\,2}} \qquad ...(1)$$

The direction of the net velocity of the swimmer is given by

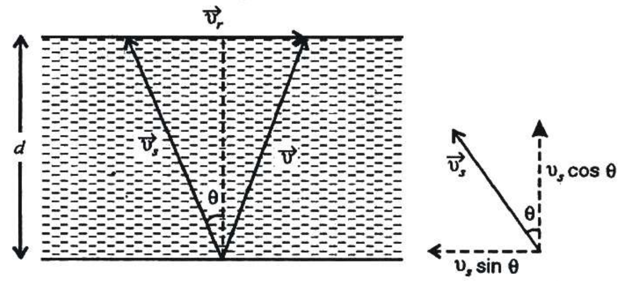
$$\tan\theta = \frac{v_r}{v_s} \qquad ...(2)$$

Time taken by the swimmer to cross the river of width d is given by

$$t = \frac{d}{v_s} \qquad ...(3)$$

Distance travelled by the swimmer down the stream, $BC = v_r \times t = \frac{v_r d}{v_s}$ ...(4)

2. Let us suppose the swimmer begins to swim at angle θ with the normal to the flow of river as shown in figure.



The component of swimmer's velocity perpendicular to the flow of river, $v_y = v_s \cos\theta$

The component of swimmer's velocity opposite to the flow of river, $v_x = v_s \sin\theta$

$\therefore$ Net speed of the swimmer along the flow of river, $v_1 = v_r - v_s \sin\theta$

Net speed of the swimmer in a direction perpendicular to the flow of river, $v_2 = v_s \cos\theta$

$\therefore$ Speed of the swimmer to cross the river,

$$|\vec{v}| = \sqrt{v_1^2 + v_2^2} = \sqrt{(v_r - v_s \sin\theta)^2 + v_s^2 \cos^2\theta}$$

$$= \sqrt{v_r^2 + v_s^2 - 2v_r v_s \sin\theta}$$

Time taken by the swimmer to cross the river, $t = \dfrac{d}{v_2} = \dfrac{d}{v_s \cos\theta}$

Distance travelled by the swimmer down the stream $= v_1 t = (v_r - v_s \sin\theta) \times \dfrac{d}{v_s \cos\theta}$

SOLVED EXAMPLE

Example 1. A motor boat traveling 4 m/s, East encounters a current traveling 7.0 m/s, North.

(A) What is the resultant velocity of the motor boat?

(B) If the width of the river is 80 meters wide, then how much time does it take the boat to travel shore to shore?

(C) What distance down stream does the boat reach the opposite shore?

Solution. **(A)** The resultant velocity can be found using the Pythagorean theorem. The resultant is the hypotenuse of a right triangle with sides of 4 m/s and 7 m/s. It is

$$V = \sqrt{(4m/s)^2 + (7m/s)^2} = 8.06\ m/s$$

Its direction can be determined using a trigonometric function.

Direction = $\tan^{-1}$ [(7 m/s) / (4 m/s)] = 60°

(B) The time to cross the river is t = d / v = (80 m) / (4 m/s) = 20 s

(C) The distance traveled downstream is d = v • t = (7 m/s) • (20 s) = 140 m

Example 2. A plane can travel with a speed of 80 m/hr with respect to the air. Determine the resultant velocity of the plane (magnitude only) if it encounters a

(A) 10 m/hr headwind.

(B) 10 m/hr tailwind.

(C) 10 m/hr crosswind.

(D) 60 m/hr crosswind.

Solution. (A) A headwind would decrease the resultant velocity of the plane to 70 m/hr.

(B) A tailwind would increase the resultant velocity of the plane to 90 m/hr.

(C) A 10 mi/hr crosswind would increase the resultant velocity of the plane to 80.6 m/hr.

This can be determined using the Pythagorean theorem:

$$V = \sqrt{(80m/hr)^2 + (10m/hr)^2}$$

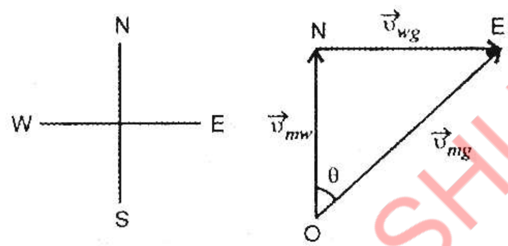
(D) A 60 mi/hr crosswind would increase the resultant velocity of the plane to 100 m/hr.

This can be determined using the Pythagorean theorem:

$$V = \sqrt{(80m/hr)^2 + (60m/hr)^2}$$

Example 3. A man swims across the river which flows with a velocity of 3 km h^{-1} due east. If the velocity of man relative to water is 4 km h^{-1} due north, then (i) what is his velocity relative to the shore of the river ? (ii) How long does he take to cross the river if river is 1 km wide? (iii) How far from his starting point will he reach the opposite bank ?

Solution.



Here $\vec{v}_{mw} = 4 kmh^{-1}$ due north is the velocity of the man w.r.t. water.

$\vec{v}_{wg} = 3 kmh^{-1}$ due east is the velocity of water in river w.r.t. ground.

Let $\vec{v}_{mg}$ be the velocity of man w.r.t. the ground or shore. Draw the vectors as shown in figure

(i) $\therefore \ \vec{v}_{mg} = \vec{v}_{mw} + \vec{v}_{wg}$

(Triangle law of vector addition)

Magnitude of $\vec{v}_{mg}$ is given

$$v_{mg} = \sqrt{v_{mw}^2 + v_{mg}^2}$$

$$= \sqrt{(4)^2 + (3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5 \text{ kmh}^{-1}$$

Direction of v_{mg} is given by

$$\tan\theta = \frac{v_{wg}}{v_{mw}} = \frac{3}{4} = 0.7500$$

or $\theta = \tan^{-1}(0.7500) = 36°52'$

Thus, velocity of man w.r.t. the shore is 5 km h^{-1} in the direction of $36°52'$ east of north

(ii) Time taken to cross the river,

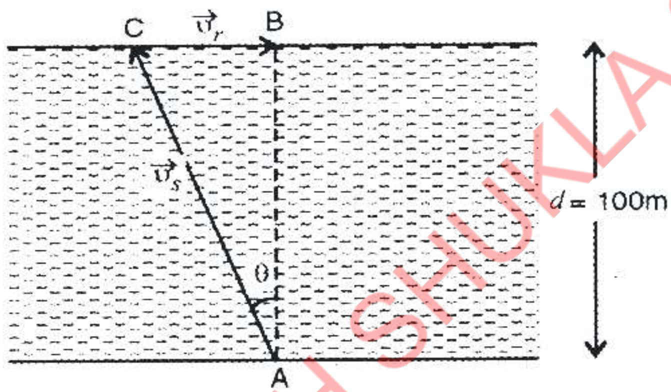
$$t = \frac{\text{Distance}}{\text{Speed of man w.r.t. water}} = \frac{1\,\text{km}}{4\,\text{kmh}^{-1}} = \frac{1}{4}\text{h} = 15\text{ min}$$

(iii) Distance travelled by man down the stream = Velocity of flow × Time

$$= 3\,\text{kmh}^{-1} \times \frac{1}{4}\text{h} = 0.75\text{ km} = 750\text{ m}$$

Example 4. A swimmer can swim with velocity of 10 km h^{-1} w.r.t. the water flowing in a river with velocity of 5 km h^{-1}. (i) In what direction should he swim to reach the point on the other bank just opposite to his starting point ? (ii) What is the time taken by him to cross the river if it is 100 m wide ?

Solution. (1) The swimmer must swim at an angle θ with the normal to the flow of river to reach point B just opposite to point A to compensate the downward drift due to the flow of water.



The angle θ with the normal to the flow of river to reach point B just opposite to point A to compensate the downward drift due to the flow of water.

The angle θ is given by

$$\sin\theta = \frac{v_r}{v_s} = \frac{5}{10} = 0.5$$

$$\theta = \sin^{-1}(0.5) = 30°$$

Thus the swimmer must swim at an angle of 90° + 30° = 120° w.r.t. the direction of the flow.

(ii) Now component of $\vec{v}_s$ along AB,

$$v = v_s\cos\theta = 10 \times \cos 30° = 10 \times \frac{\sqrt{3}}{2}$$

$$= 5\sqrt{3}\,\text{km h}^{-1}$$

$$= 5\sqrt{3} \times \frac{5}{18} = \frac{25\sqrt{3}}{18} = 2.4\text{ ms}^{-1}$$

$\therefore$ time taken by him to cross the river,

$$t = \frac{d}{v} = \frac{100\,\text{m}}{2.4\,\text{ms}^{-1}} = 41.6\,\text{s}$$

Example 5. A steam boat goes across a lake and comes back : (a) on a quiet day when the water is still and (b) on a rough day when there is a uniform current so as to help the journey onward and to impede the journey backward. If the speed of launch on both days was same, in which case will it complete the journey in lesser time?

Solution. Let l be the width of lake and v be the velocity of steam boat.

On a quiet day, time taken by steam boat in going and coming back is.

$$t_Q = \frac{\ell}{v} + \frac{\ell}{v} = \frac{2\ell}{v} \qquad ...(1)$$

on a rough day, let v' be the velocity of air current. As in going across the lake, the air current helps the motion, so time taken is $t_1 = \frac{\ell}{v + v'}$

In coming back, as the air current opposes the motion, so time taken is $t_2 = \frac{\ell}{v - v'}$

Total time in going and coming back on a rough day

$$t_R = t_1 + t_2 = \frac{2\ell v}{(v^2 - v'^2)}$$

$$= \frac{2\ell}{v[1-(v'/v)^2]} \qquad ...(2)$$

from (i) and (ii), we have

$$\frac{t_R}{t_Q} = \frac{1}{[1-(v'/v')^2]} > 1 \text{ or } t_R > t_Q$$

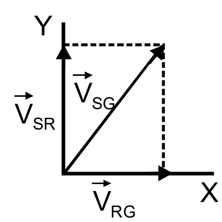
Therefore the time taken to complete the journey on quiet day is less than on a rough day.

Example 6. A swimmer can swim in still water at a rate 4 km/hour. If he swims in a river flowing at 3 km/h and keeps his direction (with respect to water.) perpendicular to the current. Find his velocity with respect to the ground.

Solution. The velocity of the swimmer with respect to water is $\vec{v}_{SR} = 4.0 \text{km/hr}$ in the direction perpendicular to the river. The velocity of river with respect to the ground is $\vec{v}_{RG} = 3.0 \text{km/hr}$ along the length of river.

The velocity of the swimmer with respect to the ground is $\vec{v}_{SG}$ where

$$\vec{V}_{SG} = \vec{V}_{SR} + \vec{V}_{RS}$$



$$V_{SG} = \sqrt{V_{SR}^2 + V_{RG}^2} = \sqrt{4^2 + 3^2}$$

$$= \sqrt{16+9} = \sqrt{25} = 5 \text{km/hr}$$

The angle q made with the direction of flow is

$$q = \tan^{-1}\left[\frac{V_{SR}}{V_{RG}}\right] = \tan^{-1}\left(\frac{4}{3}\right)$$

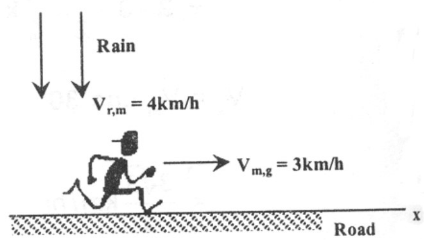
EXERCISE

1. A man can swim in still water at a speed of 3 km/hr. He wants to, cross a 500 m wide river flowing at 2 km/hour. He keeps himself always at an angle of 120^0 with the river flow while swimming.

(a) Find the time he takes to cross the river.

(b) At what point on the opposite bank will he arrive.

2. A man is walking on a level road at a speed of 3 km/hr. Raindrops fall vertically with a speed of 4 km/hr. Find the velocity of raindrops with respect to the men.



3. A girl standing on a road has to hold her umbrella at 30^0 with the vertical to keep the rain away. She throws the umbrella and starts running at 10 km/hr. She finds that raindrops are hitting her head vertically. Find the speed of raindrops with respect to (a) the road (b) the moving girl.

4. The driver of a train travelling at 115 km/hour sees on the same track 100 m infront of him a slow train travelling in same direction at 25 km/hr. The least retardation that must be applied to the faster train to avoid a collision will be

(1) 31.25 m/s^2 (2) 3.125 m/s^2

(3) 312.5 m/s^2 (4) 0.3125 m/s^2

5. A motor boat covers a given distance in 6 hour moving down stream of a river. It covers the same distance in 10 hours moving upstream. What time would it take to cover the same distance in still water ?

6. A motor boat traveling 6 m/s, East encounters a current traveling 3.8 m/s, South.

(A) What is the resultant velocity of the motor boat?

(B) If the width of the river is 120 meters wide, then how much time does it take the boat to travel shore to shore?

(C) What distance downstream does the boat reach the opposite shore?

7. A lady walking due east on a road with velocity of 10 ms^{-1} encounters rain falling vertically with a velocity of \30 ms^{-1}. At what angle she should hold her umbrella to protect herself from the rain?

(Ans. θ = 18°26′)

8. If the current velocity in question 3 were increased to 5 m/s, then

(A) How much time would be required to cross the same 120-m wide river?

(B) What distance downstream would the boat travel during this time?

(Ans. (A) 20 s, (B) 100 m.)

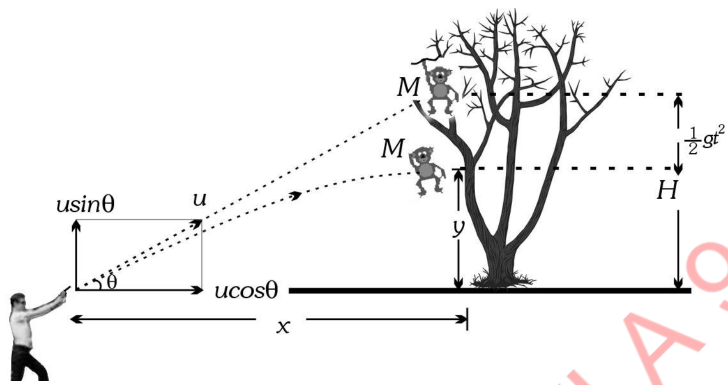
9. Wind is blowing at speed of 70 km h^{-1} and the flag hoisted on a ship in a harbour flutters along N-E direction. If the ship starts moving at a speed of 50 km h^{-1} toward north, what will be the direction of the flag hoisted on the ship ? **(Ans. α = 45.5°)**

10. A load of 5 kg is suspended by a metallic string from a fixed support. A force F is applied on the load in a horizontal direction to displace the load through an angle of 60º from its rest position. Find the value of F. **(Ans. 84.87 N)**

TYPES OF CURVILINEAR MOTIONS

1. Projectile Motion: motion of a particle under the effect of gravity.
2. Circular Motion: motion of a particle along a circle.

A hunter aims his gun and fires a bullet directly towards a monkey sitting on a distant tree. If the monkey remains in his position, he will be safe but at the instant the bullet leaves the barrel of gun, if the monkey drops from the tree, the bullet will hit the monkey because the bullet will not follow the linear path.



The path of motion of a bullet will be parabolic and this motion of bullet is defined as projectile motion.

Projectile

It is a name given to a body which is thrown with some initial velocity and then allowed to move under the action of gravity alone.

Example

(i) A bomb released from an aeroplane in level flight

(ii) A bullet fired from a gun

(iii) An arrow released from bow

(iv) A Javelin thrown by an athlete

Projectile Motion

When a body moves under an acceleration whose direction is different from the direction of the initial velocity, then both the magnitude and the direction of its velocity changes with time. Hence the body moves on a curved path in a plane. This type of motion is called 'plane motion' and the best example of this type of motion is "Projectile Motion".

Assumptions of Projectile Motion

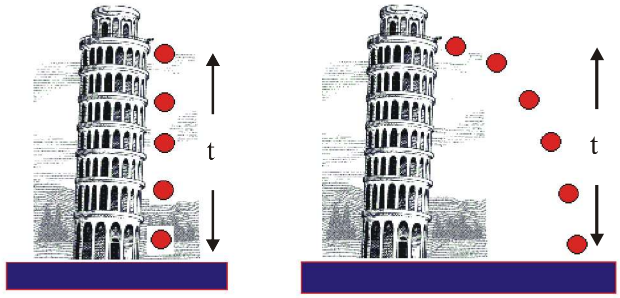
(1) There is no resistance due to air.

(2) The effect due to curvature of earth is negligible.

(3) The effect due to rotation of earth is negligible.

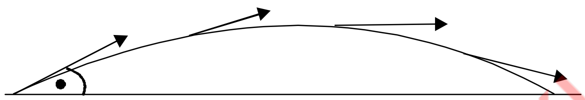
(4) For all points of the trajectory, the acceleration due to gravity 'g' is constant in magnitude and direction.

The projectile motion can always be resolved into two simple motions; one horizontal and the other vertical motion. But these two motions are completely independent of each other. The horizontal velocity given to the projectile is not affected by the vertical velocity created due to 'g'. Therefore, if we drop down a ball from the top of a tower and at the same instant throw another ball in horizontal direction, then both the balls would strike the earth simultaneously at different places.



Such a particle will move horizontally and as well as vertically i.e. along a curve.

- The velocity of particle at any instant is directed along the tangent to the path and can have horizontal and vertical components.



- The only force acting on the particle is its weight (mg) directed downward and hence acceleration due to gravity is 'g' directed vertically downwards.
- As acceleration does not change with time, the projectile motion is a uniformly accelerated motion. At all time instants, $a_x = 0$ and $a_y = -g$.

Principles of Physical Independence of Motions

(1) The motion of a projectile is a two-dimensional motion. So, it can be discussed in two parts. Horizontal motion and vertical motion. These two motions take place independent of each other. This is called the principle of physical independence of motions.

(2) The velocity of the particle can be resolved into two mutually perpendicular components. Horizontal component and vertical component.

(3) The horizontal component remains unchanged throughout the flight. The force of gravity continuously affects the vertical component.

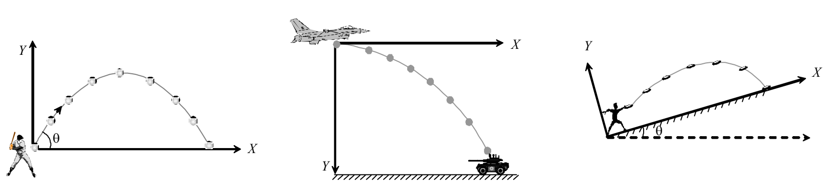
(4) The horizontal motion is a uniform motion and the vertical motion is a uniformly accelerated or retarded motion.

Types of Projectile Motion.

(1) Oblique projectile motion

(2) Horizontal projectile motion

(3) Projectile motion on an inclined plane



Analysis of Motion of A Particle as Projectile

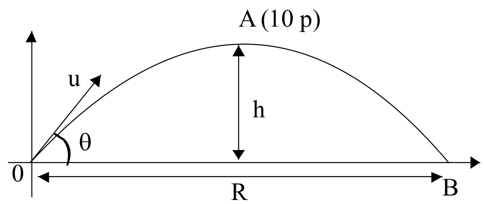
(1) Oblique Projectile Motion

Consider a particle thrown with an initial velocity **u** at an angle above the horizontal from the ground. It goes up and reaches the maximum height at **A** and then comes back to ground at **B**.

let, h = maximum height attained

R = range OB = net horizontal displacement

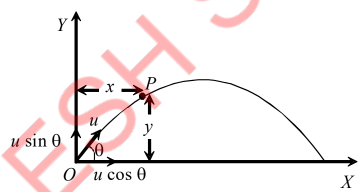
T = time of flight = total time for which it was in air



In projectile motion, horizontal component of velocity (u cosθ), acceleration (g) and mechanical energy remains constant while, speed, velocity, vertical component of velocity (u sin θ), momentum, kinetic energy and potential energy all changes. Velocity, and KE are maximum at the point of projection while minimum (but not zero) at highest point.

(i) Equation of trajectory: A projectile thrown with velocity **u** at an angle θ with the horizontal. The velocity **u** can be resolved into two rectangular components.

u cos θ component along X–axis and **u sin** θ component along Y–axis.



The equations v = u + at, s = ut + 1/2 at^2 and $v^2 = u^2 + 2as$ will be applied separately to horizontal and vertical motions.

For horizontal motion u= u cosθ and a=0

$x = u\cos\theta \times t \quad \Rightarrow \quad t = \dfrac{x}{u\cos\theta}$ .... (i)

For vertical motion u= u sin θ and a=-g

$y = (u\sin\theta)t - \dfrac{1}{2}gt^2$ .... (ii)

From equation (i) and (ii) $y = u\sin\theta\left(\dfrac{x}{u\cos\theta}\right) - \dfrac{1}{2}g\left(\dfrac{x^2}{u^2\cos^2\theta}\right)$

$$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$$

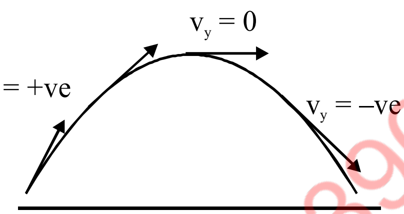
This equation shows that the trajectory of projectile is parabolic because it is similar to equation of parabola

$y = ax - bx^2$

NOTE: Equation of oblique projectile also can be written as

$$y = x\tan\theta\left[1 - \frac{x}{R}\right] \qquad \text{(where R = horizontal range = } \frac{u^2 \sin 2\theta}{g}\text{)}$$

(ii) Time of Flight :



$$s_y = u_y t + \frac{1}{2} a_y t^2$$

$$\Rightarrow 0 = (u\sin\theta)T + \frac{1}{2}(-g)T^2$$

$$T = \frac{2u\sin\theta}{g}$$

(ii) Range :

$S_x = u_x$ x T = Range

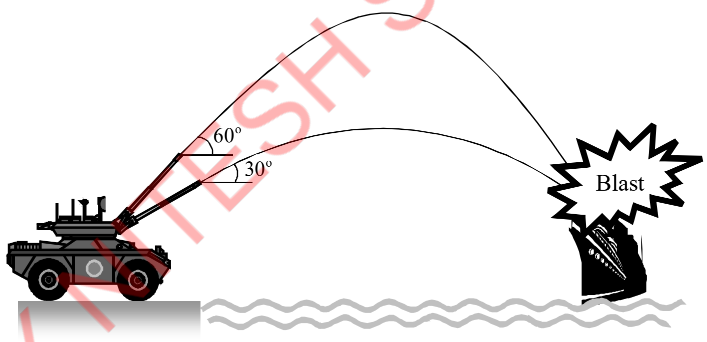
$$R = (u\cos\theta)\left(\frac{2u\sin\theta}{g}\right)$$

$$\Rightarrow R = \frac{2u\sin 2\theta}{g}$$

At the topmost point, the tangent to the path is horizontal and hence velocity vector is horizontal.

$v_y = 0$ at the topmost point.

(i) If angle of projection is changed from θ to $\theta' = (90° - \theta)$ then range remains unchanged.



$$R' = \frac{u^2 \sin 2\theta'}{g} = \frac{u^2 \sin[2(90° - \theta)]}{g} = \frac{u^2 \sin 2\theta}{g} = R$$

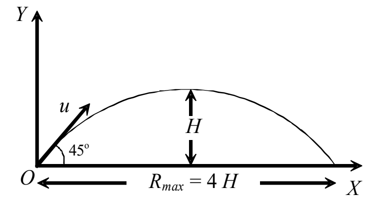
So a projectile has same range at angles of projection θ and $(90° - \theta)$, though time of flight, maximum height and trajectories are different.

These angles θ and $90^o - \theta$ are called complementary angles of projection and for complementary angles of projection ratio of range $\frac{R_1}{R_2} = \frac{u^2 \sin\sin 2\theta / g}{u^2 \sin[2(90° - \theta)]/g} = 1 \Rightarrow$

(ii) For angle of projection $\theta_1 = (45° - \alpha)$ and $\theta_2 = (45° + \alpha)$, range will be same and equal to $u^2 \cos 2\alpha/g$.

θ_1 and θ_2 are also the complementary angles.

(iii) Maximum range : For range to be maximum



$$\frac{dR}{d\theta} = 0 \Rightarrow \frac{d}{d\theta}\left[\frac{u^2 \sin 2\theta}{g}\right] = 0$$

$\Rightarrow$ cos 2θ = 0 i.e. 2θ = 90° $\Rightarrow$ θ = 45° and $R_{max} = (u^2/g)$

i.e., a projectile will have maximum range when it is projected at an angle of 45° to the horizontal and the maximum range will be (u^2/g).

When the range is maximum, the height H reached by the projectile

$$H = \frac{u^2 \sin^2\theta}{3g} = \frac{u^2 \sin^2 45}{2g} = \frac{u^2}{4g} = \frac{R_{max}}{4}$$

i.e., if a person can throw a projectile to a maximum distance R_{max}, The maximum height to which it will rise is $\left(\frac{R_{max}}{4}\right)$.

(iv) Relation between horizontal range and maximum height : $R = \frac{u^2 \sin 2\theta}{g}$ and $H = \frac{u^2 \sin^2\theta}{2g}$

$$\therefore \frac{R}{H} = \frac{u^2 \sin 2\theta / g}{u^2 \sin^2\theta / g} = 4\cot\theta \qquad \Rightarrow R = 4H\cot\theta$$

(v) If in case of projectile motion range **R** is **n** times the maximum height **H**

i.e. $R = nH \quad \frac{u^2 \sin 2\theta}{g} = n\frac{u^2 \sin^2\theta}{2g} \Rightarrow \tan\theta = [4/n] \quad \Rightarrow$ or $\theta = \tan^{-1}[4/n]$

The angle of projection is given by $\theta = \tan^{-1}[4/n]$

Note : If R = H then or .$\theta = \tan^{-1}(4)$ or θ = 76°

If R = 4H then or $\theta = \tan^{-1}(1)$ or θ = 45°.

1. Maximum Height above the Ground :

$v^2 - u^2 = 2as$

For y – axis

$v_y^2 = u_y^2 2a_y s_y$ (a = $-a_y$ inpuward motion)

$$h = \frac{u^2 \sin^2\theta}{2g} = \frac{{u_y}^2}{2g}$$

Change in momentum: Simply by the multiplication of mass in the above expression of velocity

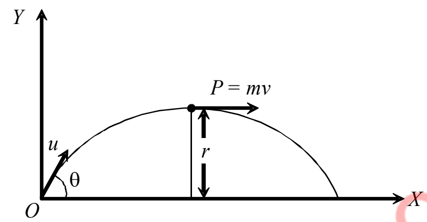
(i) Change in momentum (Between projection point and highest point) $\Delta p = \vec{p}_f - \vec{p}_i = -mu\sin\theta\,\hat{j}$

(ii) Change in momentum (For the complete projectile motion) $\Delta p = \vec{p}_f - \vec{p}_i = -2mu\sin\theta\,\hat{j}$

Angular momentum : Angular momentum of projectile at highest point of trajectory about the point of projection is given by

$$L = mvr \left[\text{Here } r = H = \frac{u^2 \sin^2\theta}{2g}\right]$$

$$\therefore \quad L = m\,u\cos\theta \frac{u^2 \sin^2\theta}{2g} = \frac{m\,u^3 \cos\theta \sin^2\theta}{2g} \;\therefore$$



Change in velocity : Initial velocity (at projection point)

Final velocity (at highest point) $\vec{u}_i = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

(i) Change in velocity (Between projection point and highest point) $\Delta u = \vec{u}_f - \vec{u}_i = -u\sin\theta\,\hat{j}$

When body reaches the ground after completing its motion then final velocity $\vec{u}_f = u\cos\theta\,\hat{i} - u\sin\theta\,\hat{j}$

(ii) Change in velocity (Between complete projectile motion) $\Delta u = u_f - u_i = -2u\sin\theta\,\hat{i}$

Resultant velocity (velocity of projectile at any time)

Along x – axis

$v_x = u_x + a_x t$

Here $u_x = u$ & $a_x = 0$

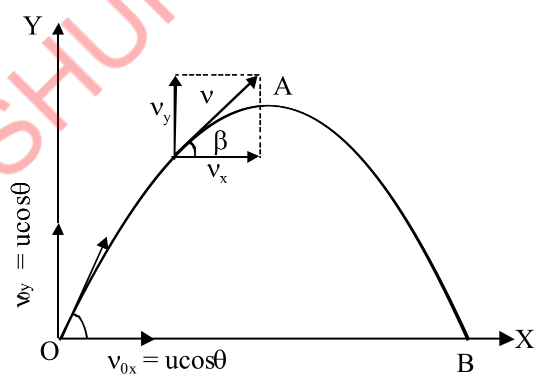
$\therefore\; v_x = u$

Along y – axis

$v_y = u_y + a_y t$

Here $u_y = 0$ & $a_y = g$

$\therefore\; u_y = gt$



As both v_x & v_y are $\perp$ to each other

Resultant at time t = v(t)

$$v(t) = \sqrt{v_x^2 + v_y^2 + 2v_x y_y \cos 90^\circ}$$

$$v(t) = \sqrt{v_x^2 + v_y^2}$$

If resultant velocity makes an angle β with the horizontal direction

$$\tan = \frac{v_y}{v_x} = \frac{gt}{u}$$

$$\beta = \tan^{-1} = \left(\frac{gt}{u}\right)$$

Velocity of the projectile when it hits the ground:

Along x – axis

$v_x = u2gh_x + a_x t$ Here $u_x = u$ & $a_x = 0$

$\therefore \quad v_x = u$

Along y axis

$v_y = u_y + a_y t$ Here $u_y = 0$ & $a_y = g$

$$\therefore \quad v_y = gt = g\sqrt{\frac{2h}{g}} = \sqrt{2gh}$$

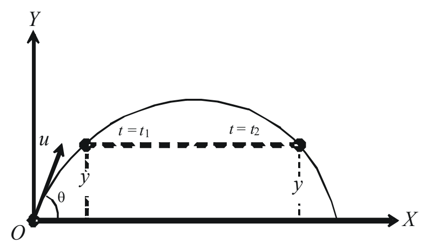
$$v(t) = \sqrt{v_x^2 + v_y^2}$$

$$\therefore \quad v(t) = \sqrt{u^2 + 2gh}$$

Vertical component of velocity at any time:

- is zero, if the particle moving horizontally (at the highest point)
- is +ve if it is going up.
- is -ve if it is coming down.

Projectile passing through two different points on same height at time t_1 and t_2 : If the particle passes two points situated at equal height *y* at $t = t_1$ and $t = t_2$, then



(i) Height (*y*): $y = (u\sin\theta)t_1 - \frac{1}{2}gt_1^2$ .....(i)

and $y = (u\sin\theta)t_2 - \frac{1}{2}gt_2^2$ .....(ii)

Comparing equation (i) with equation (ii)

$$u\sin\theta = \frac{g(t_1 + t_2)}{2}$$

Substituting this value in equation (i)

$$y = g\left(\frac{t_1 + t_2}{2}\right)t_1 - \frac{1}{2}gt_1^2 \Rightarrow y = \frac{gt_1t_2}{2}$$

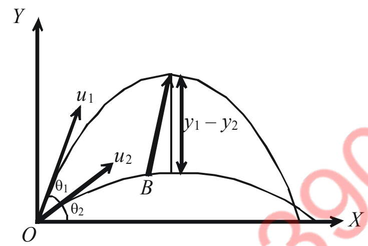
(ii) Time (t_1 and t_2): $y = u\sin\theta t - \frac{1}{2}gt^2$

$$t^3 - \frac{2u\sin\theta}{g}t + \frac{2y}{g} = 0 \Rightarrow t = \frac{u\sin\theta}{g}\left[1 \pm \sqrt{1 - \left(\frac{\sqrt{2gy}}{u\sin\theta}\right)^2}\right]$$

$$t_1 = \frac{u\sin\theta}{g}\left[1 + \sqrt{1 - \left(\frac{\sqrt{2gy}}{u\sin\theta}\right)}\right] \text{ and } t_2 = \frac{u\sin\theta}{g}\left[1 - \sqrt{1 - \left(\frac{\sqrt{2gy}}{u\sin\theta}\right)^2}\right]$$

Motion of a projectile as observed from another projectile : Suppose two balls *A* and *B* are projected simultaneously from the origin, with initial velocities u_1 and u_2 at angle θ_1 and $_2$, respectively with the horizontal.

The instantaneous positions of the two balls are given by



Ball *A* : $x_1 = (u_1 \cos\theta_1)t$ $y_1 = (u_1 \sin\theta_1)t - \frac{1}{2}gt^2$

Ball *B* : $x_2 = (u_2 \cos\theta_2)t$ $y_2 = (u_2 \sin\theta_2)t - \frac{1}{2}gt^2$

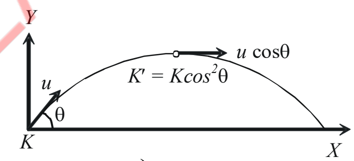
The position of the ball *A* with respect to ball *B* is given by

$x = x_1 - x_2 = (u_1 \cos\theta_1 - u_2 \cos\theta_2)t$

$y = y_1 - y_2 = (u_1 \sin\theta_1 - u_2 \sin\theta_2)t$

Now $\frac{y}{x} = \left(\frac{u_1 \sin\theta_1 - u_2 \sin\theta_2}{u_1 \cos\theta_1 - u_2 \cos\theta_2}\right) = \text{constant}$

If a body is projected with initial kinetic energy K(=1/2 mu^2), with angle of projection θ with the horizontal then at the highest point of trajectory



(i) **Kinetic energy** $= \frac{1}{2}m(u\cos\theta)^2 = \frac{1}{2}mu^2\cos^2\theta$

$k' = k\cos^2\theta$

(ii) **Potential energy** $= mgH = mg\frac{u^2\sin^2\theta}{2g} = \frac{1}{2}mu^2\sin^2\theta$ $\left(\text{As } H = \frac{u^2\sin^2\theta}{2g}\right)$

(iii) **Total energy** = Kinetic energy + Potential energy $= \frac{1}{2}mu^2\cos^2\theta + \frac{1}{2}mu^2\sin^2\theta$

$= \frac{1}{2}mu^2$ = Energy at the point of projection.

This is in accordance with the law of conservation of energy.

SOLVED EXAMPLE

Example 7. The trajectory of a projectile is represented by $y = \sqrt{3}x - gx^2/2$. What is the angle of projection?

Solution : By comparing the coefficient of x in given equation with standard equation

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$\tan\theta = \sqrt{3}$ $\therefore\ \theta = 60°$

Example 8. The path followed by a body projected along y-axis is given as by $y = \sqrt{3}x - (1/2)x^2$, if g = 10 m/s, then the initial velocity of projectile will be – (x and y are in m)

(A) $3\sqrt{10}$ m/s (B) $2\sqrt{10}$ m/s

(C) $10\sqrt{3}$ m/s (D) $10\sqrt{2}$ m/s

Solution : By comparing the coefficient of x^2 in given equation with standard equation

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}.$$

$$\frac{g}{2u^2\cos^2\theta} = \frac{1}{2}$$

Substituting = 60° we get $u = 2\sqrt{10}\,m/sec$.

Example 9. A body of mass 2 kg has an initial velocity of 3 m/s along OE and it is subjected to a force of 4 Newton's in OF direction perpendicular to OE. What will be the distance of the body from O after 4 seconds.

Solution : Body moves horizontally with constant initial velocity 3 m/s upto 4 seconds

$$\therefore \quad x = ut = 3 \times 4 = 12m$$

and in perpendicular direction it moves under the effect of constant force with zero initial velocity upto 4 seconds.

$$\therefore \quad y = ut + \frac{1}{2}(a)t^2 = 0 + \frac{1}{2}\left(\frac{F}{m}\right)t^2 = \frac{1}{2}\left(\frac{4}{2}\right)4^2 = 16m$$

So its distance from O is given by

$$d = \sqrt{x^2 + y^2} = \sqrt{(12)^2 + (16)^2}$$

$$\therefore \quad d = 20\,m$$

Example 10. In a projectile motion, velocity at maximum height is

(A) $\frac{u\cos\theta}{2}$ (B) $u\cos\theta$

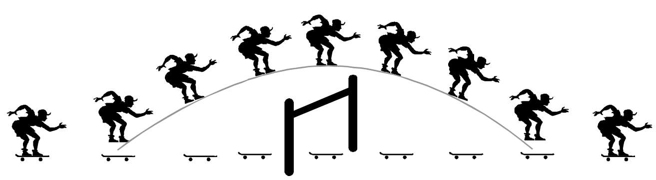
(C) $\frac{u\sin\theta}{2}$ (D) None of these

Solution : **(b)**

In a projectile motion at maximum height body possess only horizontal component of velocity i.e. u cosθ.

Example 11. When a man jumps over the hurdle leaving behind its skateboard then vertical component of his velocity is changing, but not the horizontal component, which matches with the skateboard velocity.

As a result, the skateboard stays underneath him, allowing him to land on it.



Let v_i be the instantaneous velocity of projectile at time t direction of this velocity is along the tangent to the trajectory at point P.

$$\vec{v}_i = v_x i + v_y \hat{j} \Rightarrow v_i = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2\cos^2\theta + (u\sin\theta - gt)^2}$$

$$v_i = \sqrt{u^2 + g^2t^2 - 2ugt\sin\theta}$$

Direction of instantaneous velocity $\tan\alpha = \frac{v_y}{v_x} = \frac{u\sin\theta - gt}{u\cos\theta}$ or $\alpha = \tan^{-1}\left[\tan\theta - \frac{gt}{u}\sec\theta\right]$

Example 12. A stone is thrown with a velocity of 19.6 m/s at an angle of 30° above horizontal from the top of a building 14.7m high. Find:

(i) The time after which the stone strikes the ground.

(ii) The distance of the landing point of the stone from the building.

(iii) The velocity with which the stone hits the ground.

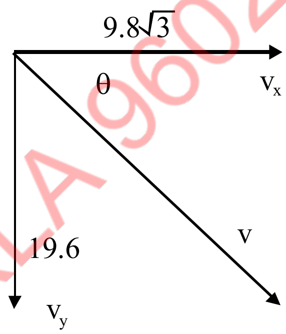
(iv) The maximum height attained by the stone above the ground.

Solution. Consider the interval from O to C

u_x = 19.6 x cos 30° = 9.83 m/s ; a_x = 0 m/s^2

u_y = 19. 6 x sin 30° = 9. 8 m/s ; a_y = – 9.8 m/s^2

Along vertical direction : s_y = – 14.7 m



(a) $s_y = u_y t + \frac{1}{2} a_y t^2$

$-14.7 = 9.8\,t + \frac{1}{2}(-9.8)\,t^2$

$\Rightarrow 4.9\,t^2 - 9.8\,t - 14.7 = 0$

$\Rightarrow t = -1, 3s$

$\Rightarrow$ Stone lands at see after 3 seconds

(b) From 0 to C, the horizontal displacement = S

$S_x = u_x t$ = (19.6 cos30°) x3 = 50.92 m

Distance of C from the building = AC = 50.92m.

(c) The horizontal velocity remains constant.

Hence at C, $v_x = u_x = 9.8\sqrt{3}$ m/s

$v_y + u_y + a_y t$

v_y + 19.6x sin30^o – 9.8 x^3

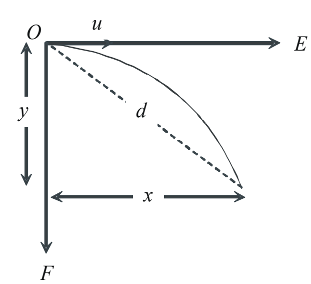
v_y – 19.6 m/s.

v_y is – v_y because the stone is moving down when it hits the ground.

Resultant velocity

$\sqrt{v_x^2 + v_y^2}$

$\sqrt{(9.8\sqrt{3^2}) + (19.6)^2} = -9.8\sqrt{7}$ m/s



Velocity is directed at an angle θ given by :

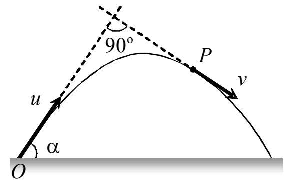
$\theta = \tan{-1}\left[\frac{v_y}{v_x}\right] = \tan^{-1}\left[\frac{19.6}{9.8\sqrt{3}}\right]$

$= \tan{-1}\left[\frac{1}{\sqrt{3}}\right]$ below horizontal

(d) Maximum height attained above ground = height of B above point of projection of height of building = h + 14.7 m

= $(u^2/2g)\sin^2 30^o$ + 14.7 = 19.6 m

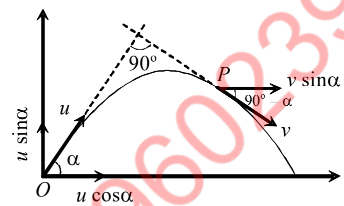
Example 13. A particle is projected from point O with velocity u in a direction making an angle with the horizontal. At any instant its position is at point P at right angles to the initial direction of projection. What is its velocity at point P?



Solution : Horizontal velocity at point

Horizontal velocity at point

In projectile motion horizontal component of velocity remains constant throughout the motion



$\therefore \Rightarrow$ v sin α = u cos α ⇒ v = u cot α

Example 14. The range R of projectile is same when its maximum heights are h_1 and h_2. What is the relation between R and h_1 and h_2?

Solution. For equal ranges body should be projected with angle or from the horizontal.

And for these angles : $h_1 = \dfrac{u^2 \sin^2\theta}{2g}$ and $h_2 = \dfrac{u^2\cos^2\theta}{2g}$

by multiplication of both height : $h_1 h_2 = \dfrac{u^2\sin^2\theta\cos^2\theta}{4g^2} = \dfrac{1}{16}\left(\dfrac{u^2\sin 2\theta}{g}\right)^2$

$\Rightarrow$ $16\, h_1 h_2 = R^2$

$\Rightarrow$ $R = 4\sqrt{h_1 h_2}$

Example 15. A projectile is projected with initial velocity $(6\hat{i} + 8\hat{j})\,m/sec.$ If $g = 10\ ms^{-2}$, then what is the horizontal range of the projectile?

Solution. Initial velocity $= (6\hat{i} + 8\hat{j})\,m/s$ (given)
Magnitude of velocity of projection = $u = \sqrt{u_x^2 + u_y^2} = \sqrt{6^2 + 8^2}$ 10 m/s

Angle of projection $\tan\theta = \dfrac{u_y}{u_x} = \dfrac{8}{6} \therefore \sin\theta = \dfrac{4}{5}$ and $\cos\theta = \dfrac{3}{5}$

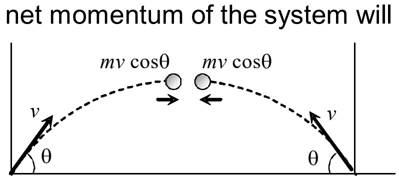
Now horizontal range $R = \dfrac{u^2\sin 2\theta}{g} = \dfrac{u^2 2\sin\theta\cos\theta}{g} = \dfrac{(10)^2 \times 2 \times \frac{4}{5} \times \frac{3}{5}}{10} = 9.6\,meter$

Example 16. Two equal masses (m) are projected at the same angle (θ) from two points separated by their range with equal velocities (v). The momentum at the point of their collision is

(a) Zero
(b) 2 mv cosθ
(c) – 2 mv cosθ
(d) None of these

Solution. **(a)**

Both masses will collide at the highest point of their trajectory with equal and opposite momentum. So net momentum of the system will be zero.



EXERCISE

11. The equation of projectile is $y = 16x - \frac{5x^2}{4}$. The horizontal range is

(a) 16 m (b) 8 m

(c) 3.2 m (d) 12.8 m **(Ans . (d))**

12. A rifle with a muzzle velocity of 100 m/s shoots a bullet at a small target 30 m away in the same horizontal line. How high above the target must the gun be aimed so that the bullet will hit the target? **(Ans. 45 cm)**

13. A projectile shot at an angle of 45° above the horizontal strikes a building 30 m away at a point 15 m above the point of projection. Find:

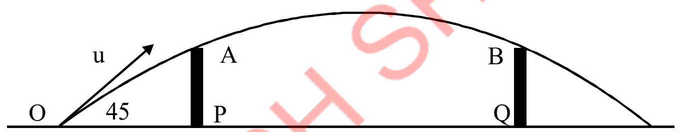
(a) the speed of projection

(b) the magnitude and direction of velocity of projectile when it strikes the building.

14. A body starts from the origin with an acceleration of 6 m/s^2 along the x-axis and 8 m/s^2 along the y-axis. What will be its distance from the origin after 4 seconds? **(Ans. 80 m)**

15. A body is thrown at angle 30° to the horizontal with the velocity of 30 m/s. What will be its velocity (in m/s) after 1 sec. (g = 10 m/s^2) **(Ans. $10\sqrt{7}$ m/s)**

16. A ball is thrown with a velocity of 7"2 m/s at an angle of 45° with the horizontal. It just clears two vertical poles of height 90 cm each. Find the separation between the poles.



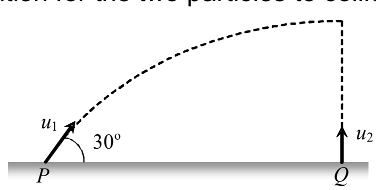
(Ans. QP = 8 m)

17. A stone is to be thrown so as to cover a horizontal distance of 3 m. If the velocity of the projectile is 7 m/s,find :

(i) the angle at which it must be thrown,

(ii) the largest horizontal displacement that is possible with the projection speed of 7 m/s.

18. A particle P is projected with velocity u_1 at an angle of 30° with the horizontal. Another particle Q is thrown vertically upwards with velocity u_2 from a point vertically below the highest point of path of P. What is the necessary condition for the two particles to collide at the highest point?



(Ans. $u_1 = 2u_2$)

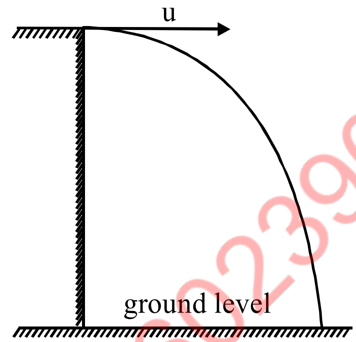
19. Two seconds after projection a projectile is travelling in a direction inclined at 30° to the horizontal after one more sec, it is travelling horizontally, find the magnitude and direction of its velocity.

(a) $2\sqrt{20}$ m/sec, 60° (b) $20\sqrt{3}$ m/sec, 60°

(c) $6\sqrt{40}$ m/sec, 30° (d) $40\sqrt{6}$ m/sec, 30° **(Ans.v = 20 m/s s & θ = 60°)**

HORIZONTAL PROJECTILE MOTION

(Projectile Thrown Horizontally from a Height)



Horizontal motion of projectile:

$u_x = u$, $a_x = 0$

$x = u\,t$

$t = \frac{x}{u}$

Vertical motion of projectile:

$u_y = 0$; $a_y = g$

$y = 1/2\ gt^2$

$y = \frac{1}{2}g\left(\frac{x}{u}\right)^2$

$y = \frac{1}{2}\left(\frac{g}{u^2}\right)x^2 = kx^2$

Time of Flight

Range :

$x = x = u_x t + \frac{1}{2}a_x t^2$

$x = R$, $t = T = \sqrt{\frac{2h}{g}}$, $u_x = u$, $a_x = 0$

$R = u_x \times T = \text{Range} \Rightarrow R = u\sqrt{\frac{2h}{g}}$

SOLVED EXAMPLE

Example 17. A block slides off a horizontal table top 1 m high with a speed of 3 m/s. Find :

(a) The horizontal distance from the edge of the table at which the block strikes the floor:

(b) the horizontal and vertical components of its velocity when it reaches the floor.

Solution: In the interval from O to B,

$u_x = 3$ m/s

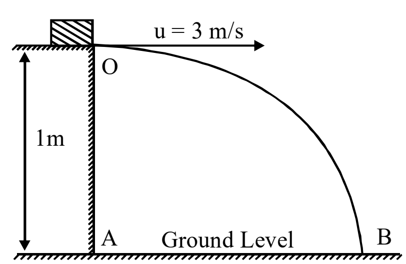
$u_y = 0$ m/s

$s_y = -1$ m As the initial velocity is horizontal, vertical component = 0 m/s.

$s_y = u_y\ t + 1/2\ a_y\ t^2$

$-1 = 0\ (t) + 1/2\ (-g)\ t^2$

$\Rightarrow\ t = \frac{\sqrt{2}}{g} = \frac{\sqrt{2}}{7}$ s



(a) $S_x + u_x t = 3\,\frac{\sqrt{10}}{7}$ m = AB = horizontal, distance

(b) $u_x - v_x = 3$ m/s

$\Rightarrow$ $v_y = u_y + a_y t = 0 - 9.8$

$\Rightarrow$ $v_y = 1.4$ m/s

Horizontal component = $u_x = 3$ m/s = v_x

Vertical component = $v_y = -1.4\sqrt{10}$ m/s

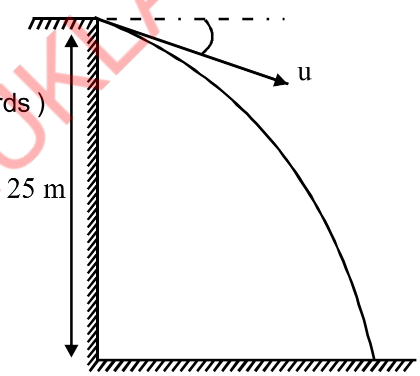
Example 18. A projectile is given an initial velocity of 5 m/s at an angle 30° below horizontal from the top of a building 25 m high. Find :

(i) the time after which it hits the ground.

(ii) the distance from the building where it strikes the ground. $g = 10$ m/s^2

Solution. The projectile is thrown from O and lands at A on the ground.

From O to A :



$s_y = -25$ m, $u_y = -5 \sin 30^0 = -2.5$ m/s

(– ve because vertical component is downwards)

(i) $a_y = -g = -10$ m/s^2

$s_y = u_y t + ½\, a_y t^2$

$-25 = -2.5\,t - 1/2(10)t^2$

So, $10t^2 + 5t - 50 = 0$

On solving, we get: t = 2s, –2.5 s

The relevant time = 2s

(ii) $s_x = u_x\, t = (5 \cos 30°)\, 2 = 5\sqrt{3}$ m.

(ii) $s_x = u_x\, t = (5 \cos 30°)\, 2 = 5\sqrt{3}$ m.

EXERCISE

21. A body is projected horizontally from the top of a cliff with a velocity of 9.8 m s^{-1}. What time elapses before the horizontal and vertical velocities become equal ? Hovem h = 9.8 m s^{-2}.(**Ans.** t = 1 s)

22. A projectile is fired horizontally with a velocity of 98 m s^{-1} from the top of the hill 490 metre high. Find

(i) The velocity with which the projectile strikes the ground

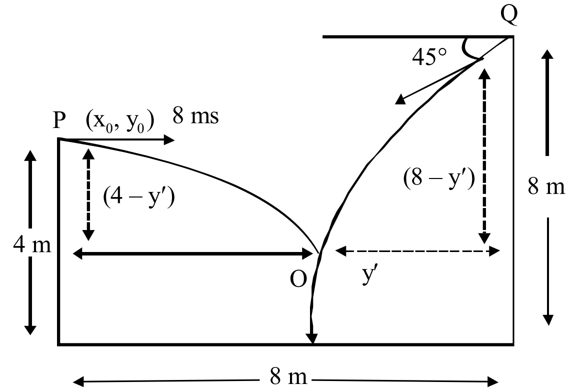
(ii) Time taken to reach the ground and

(iii) The distance of the target from the hill. **(Ans. 980 m)**

23. A ball is projected horizontally from the top of the tower of height 100 m with a velocity of 5 m s^{-1}. Calculate the time taken by the ball to reach the ground. **(Ans. t = 4.5 s)**

24. A soldier fires a bullet horizontally from the top of a cliff with a velocity of 10 ms^{-1} . If the bullet strikes the ground after 2s, find the height of the cliff. Also calculate the velocity with which the bullet strikes the ground. **(Ans. h = 19.6 m, v = 22.01 m s^{-1})**

25. Two identical balls are simultaneously thrown towards each other from points P and Q horizontally separated by 8m, and situated at heights 4 m and 8 m above the ground. One ball is thrown from P horizontally with a speed 8 m s^{-1} , while the other is thrown downward with an initial speed of v at an angle of 45º to the horizontal (figure). The two balls collide in space, calculate (a) the initial speed of ball thrown from the point Q, (b) coordinates of the point of collision. g = 10 m s^{-2}



(Ans. x'= 4m, v = 11.3 m/s, y'= 2.78 m)

26. A helicopter on flood relief mission, flying horizontally with a speed of 54 km h^{-1} at an altitude of 245 m has to drop a food packet for a victim standing on the ground. At what distance from the victim should the packet be dropped ? Take g = 10 ms^{-2}. **(Ans. x = 105 m)**

PROJECTILE MOTION ON AN INCLINED PLANE

Let a particle is thrown with a velocity u at an angle ∝ with the horizontal from the bottom of an inclined plane. Taking X-axis parallel to the plane and the Y-axis perpendicular to the plane, let us find the range and the time of flight on the inclined plane. Let the inclined plane makes an angle β with the horizontal.

Taking axis along and perpendicular to the inclined plane as shown, components of g are g cos β and g sin β as shown.

$\Rightarrow$ $a_x = -g \sin\beta$; $a_y = -g \cos \beta$

$\Rightarrow$ $u_x = u \cos(\alpha - \beta)$; $u_y = u \sin(\alpha - \beta)$

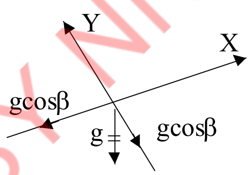


Figure (a)

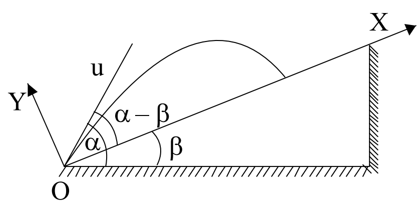


Figure (b)

From O to A

$y = 0 \Rightarrow o = u_y t \Rightarrow \frac{1}{2} \cos a_y t^2$

$\Rightarrow$ $0 = u \sin(\alpha - \beta)t - \frac{1}{2} \cos \beta t^2.$

$t = \frac{u \sin(\alpha - \beta)}{g \cos\beta}$ is the time take of flight

Substituting values of u_x, a_x and t , we get :

$$\text{Range} = \frac{2u^2 \sin(\alpha - \beta)\cos \propto}{g\cos^2 \beta}$$

'$\propto$' For Maximum Range :

Let us calculate 'α' so that the range is maximum.

R = $\dfrac{2u^2 \sin(\alpha - \beta)\cos \propto}{g\cos^2 \beta}$

For R to be maximum, $(\sin 2\alpha - \beta)$ must be maximum.

Hence $2\alpha - \beta = \dfrac{\pi}{2}$ so that R is maximum.

So, R is maximum for $\alpha = \dfrac{\pi}{4} + \dfrac{\beta}{2}$

The maximum value of R = $R_{max} = \dfrac{u^2(1-\sin\beta)}{g\cos^2 B}$

Or $R_{max} = \dfrac{u^2}{g(1+\sin\beta)}$

SOLVED EXAMPLE

Example 19. For a given velocity of projection from a point on the inclined plane, the maximum range down the plane is three times the maximum range up the incline. Then, the angle of inclination of the inclined plane is

(A) 30^o (B) 45^o

(C) 60^o (D) 90^o

Solution : **(A)**

Maximum range up the inclined plane $(R_{max})_{up} = \dfrac{u^2}{g(1+\sin\alpha)}$

Maximum range down the inclined plane $(R_{max})_{down} = \dfrac{u^2}{g(1-\sin\alpha)}$

and according to problem : $\dfrac{u^2}{g(1-\sin\alpha)} = 3 \times \dfrac{u^2}{g(1+\sin\alpha)}$

By solving $\alpha = 30^o$

Example 20. A shell is fired from a gun from the bottom of a hill along its slope. The slope of the hill is $\alpha = 30^o$, and the angle of the barrel to the horizontal $\beta = 60^o$. The initial velocity *v* of the shell is 21 *m/sec*. Then distance of point from the gun at which shell will fall

(A) 10 *m* (B) 20 *m*

(C) 30 *m* (D) 40 *m*

Solution : (C)

Here u = 21 m/sec, $\alpha = 30^o$, $\theta = \beta - \alpha = 60^o - 30^o = 30^o$

Maximum range $R = \dfrac{2u^2}{g}\dfrac{\sin\theta\cos(\theta+\alpha)}{\cos^2\alpha} = \dfrac{2\times(21)^2 \times \sin 30^o \cos 60^o}{9.8 \times \cos^2 30^o} = 30\,\text{m}$

Example 21. The maximum range of rifle bullet on the horizontal ground is 6 *km* its maximum range on an inclined of 30^o will be

(A) 1 km (B) 2 km

(C) 4 km (D) 6 km

Solution : (C)

Maximum range on horizontal plane $R = \frac{u^2}{g} = 6km$ (given)

Maximum range on a inclined plane $R_{max} = \frac{u^2}{g(1+\sin\alpha)}$

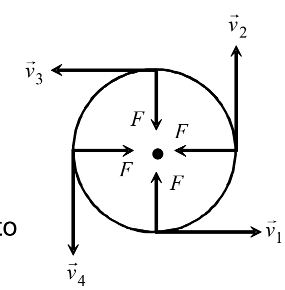
Putting $\alpha = 30^o$ $R_{max} = \frac{u^2}{g(1+\sin 30^\circ)} = \frac{2}{3}\left(\frac{u^2}{g}\right) = \frac{2}{3}\times 6 = 4km.$

EXERCISE

27. From the foot of an inclined plane, whose rise is 7 in 25, a shot is projected with a velocity of 196 m/s at an angle of 30° with the horizontal (a) uip the plane, (b) down the plane. Find the range in each case.

28. A particle projected with velocity v0, strikes at right angles a plane thruogh the point of projection and of inclination b with the horizontal. Find the height of the point struck, from horizontal plane through the point of projection.

CIRCULAR MOTION



Circular motion is another example of motion in two dimensions.

To create circular motion in a body it must have two things:

(i) some initial velocity and

(ii) A force acting on the body which is always directed at right angles to instantaneous velocity.

Since this force is always at right angles to the displacement due to the initial velocity therefore work done by the force on the particle is zero. Hence, its kinetic energy and thus speed is unaffected. But due to simultaneous action of the force and the velocity the particle follows resultant path, which is a circular.

Circular motion can be classified into two types – Uniform circular motion and non-uniform circular motion.

Variables of Circular Motion

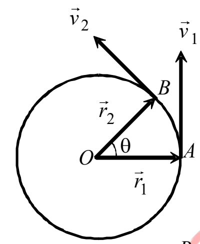
(1) **Displacement and distance :** When particle moves in a circular path describing an angle θ during time t (as shown in the figure) from the position A to the position B, we see that the magnitude of the position vector (that is equal to the radius of the circle) remains constant. i.e., $|\Delta\vec{r}_1| = |\vec{r}_2| = r$ and the direction of the position vector changes from time to time.

(i) **Displacement :** The change of position vector or the displacement $\Delta\vec{r}$ of the particle from position A to the position B is given by referring the figure.

$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

$\Rightarrow \quad \Delta r = |\Delta\vec{r}| = |\vec{r}_2 - \vec{r}_1| \quad \Delta r = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos\theta}$

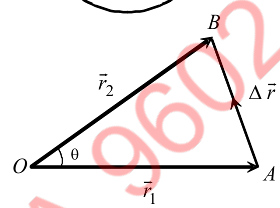
Putting $r_1 = r_2 = r$ we obtain



$$\Delta r = \sqrt{r^2 + r^2 - 2r.r.\cos\theta}$$

$$\Rightarrow \quad \Delta r = \sqrt{2r^2(1-\cos\theta)} = \sqrt{2_r^2\left(2\sin^2\frac{\theta}{2}\right)}$$

$$\Delta r = 2r\sin\frac{\theta}{2}$$



(ii) Distance : The distanced covered by the particle during the time t is given as

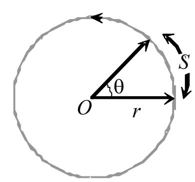
$$d = \text{length of the arc AB} = r\,\theta \quad \left(\because \quad \theta = \frac{\text{arc}}{\text{radius}}\right)$$

(iii) Ratio of distance and displacement : $\dfrac{d}{\Delta r} = \dfrac{r\theta}{2r\sin\theta/2} = \dfrac{\theta}{2}\text{cosec}(\theta/2)$

(2) Angular displacement (θ) : The angle turned by a body moving on a circle from some reference line is called angular displacement.

(i) Dimension = $[M^0L^0T^0]$ (as θ = arc/radius.

(ii) Units = Radian or Degree. It is some times also specified in terms of fraction or multiple of revolution.



(iii) 2π rad = 360° = 1 Revolution

(iv) Angular displacement is a axial vector quantity but finite angular displacement is a scalar quantity.

Its direction depends upon the sense of rotation of the object and can be given by Right Hand Rule; which states that if the curvature of the fingers of right hand represents the sense of rotation of the object, then the thumb, held perpendicular to the curvature of the fingers, represents the direction of angular displacement vector.

(v) Relation between linear displacement and angular displacement $s = \theta \times r$

or $\quad s = r\theta$

(3) Angular velocity (ω) : Angular velocity of an object in circular motion is defined as the time rate of change of its angular displacement.

(i) Angular velocity $\omega = \dfrac{\text{angle traced}}{\text{time taken}} = \lim\limits_{\Delta t \to 0}\dfrac{\Delta\theta}{\Delta t} = \dfrac{d\theta}{dt}$

$$\therefore \qquad \omega = \frac{d\theta}{dt}$$

(ii) Dimension : $[M^0L^0T^{-1}]$

(iii) Units : Radians per second ($rad.s^{-1}$) or Degree per second.

(iv) Angular velocity is an axial vector .

(v) Relation between angular velocity and linear velocity $\vec{v} = \vec{\omega} \times \vec{r}$

Its direction is the same as that of $\Delta\theta$. For anticlockwise rotation of the point object on the circular path, the direction of ω, according to Right hand rule is along the axis of circular path directed upwards. For clockwise rotation of the point object on the circular path, the direction of ω is along the axis of circular path directed downwards.

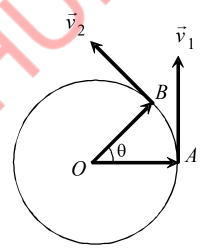
Note : It is important to note that nothing actually moves in the direction of the angular velocity vector $\vec{\omega}$. The direction of $\vec{\omega}$ simply represents that the rotational motion is taking place in a plane perpendicular to it.

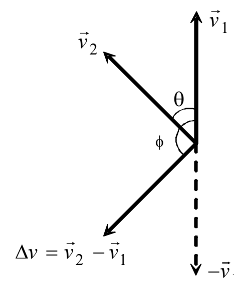
(vi) For uniform circular motion ω remains constant where as for non-uniform motion ω varies with respect to time.

(4) Change in velocity : If a particle moves from A to B during time t as shown in figure, the magnitude and direction of the change in velocity of the particle which is performing uniform circular motion is given by :

$$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$$

or $|\Delta\vec{v}| = |\vec{v}_2 - \vec{v}_1| \Rightarrow \Delta v = \sqrt{v_1^2 + v_2^2 - 2v_1v_2\cos\theta}$





For uniform circular motion $v_1 = v_2 = v$

So $|\Delta v| = \sqrt{2v^2(1-\cos\theta)} = 2v\sin\frac{\theta}{2}$

The direction of $\Delta\vec{v}$ is shown in figure that can be as

$$\phi = \frac{180^\circ - \theta}{2} = (90^\circ - \theta/2)$$

Note : Relation between linear velocity and angular velocity.

In vector form

(5) Time period (T) : In circular motion, the time period is defined as the time taken by the object to complete one revolution on its circular path.

(i) Units : second.

(ii) Dimension : $[M^0L^0T]$

(iii) Time period of second's hand of watch = 60 second.

(iv) Time period of minute's hand of watch = 60 minute

(v) Time period of hour's hand of watch = 12 hour

(6) Frequency (n) : In circular motion, the frequency is defined as the number of revolutions completed by the object on its circular path in a unit time.

(i) Units : s^{-1} or hertz (Hz).

(ii) Dimension : $[M^0L^0T^{-1}]$

Note : ❑ Relation between time period and frequency : If n is the frequency of revolution of an object in circular motion, then the object completes n revolutions in 1 second. Therefore, the object will complete one revolution in 1/n second.

$\therefore \quad T = 1/n$

❑ Relation between angular velocity, frequency and time period : Consider a point object describing a uniform circular motion with frequency n and time period T. When the object completes one revolution, the angle traced at its axis of circular motion is 2π radians. It means, when time $t = T$, $\theta = 2\pi$ radians. Hence, angular velocity $\omega = \frac{\theta}{t} = \frac{2\pi}{T} = 2\pi m$

$(\because \; T = 1/n)$

$\omega = \frac{2\pi}{T} = 2\pi m$

❑ If two particles are moving on same circle or different coplanar concentric circles in same direction with different uniform angular speeds ω_A and ω_B respectively, the angular velocity of B relative to A will be

$\omega_{rel} = \omega_B - \omega_A$

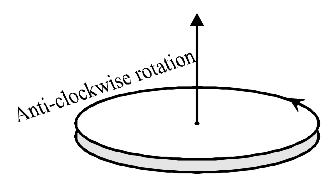
So the time taken by one to complete one revolution around O with respect to the other (i.e., time in which B complete one revolution around O with respect to the other (i.e., time in which B completes one more or less revolution around O than A)

$$T = \frac{2\pi}{\omega_{rel}} = \frac{2\pi}{\omega_2 - \omega_1} = \frac{T_1 T_2}{T_1 - T_2} \qquad \left[\text{as } T = \frac{2\pi}{\omega}\right]$$

Special case : If ω_B ω_A, $\omega_{rel} = 0$ and so $T = \infty$., particles will maintain their position relative to each other. This is what actually happens in case of geostationary satellite ($\omega_1 = \omega_2$ = constant)

(7) Angular acceleration (α) : Angular acceleration of an object in circular motion is defined as the time rate of change of its angular velocity.

(i) If $\Delta\omega$ be the change in angular velocity of the object in time interval t and $t + \Delta t$, while moving on a circular path, then angular acceleration of the object will be



$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

(ii) Units : rad. s^{-2}

(iii) Dimension : $[M^0L^0T^{-2}]$

(iv) Relation between linear acceleration and angular acceleration $\vec{a} = \vec{\alpha} \times \vec{r}$

(v) For uniform circular motion since ω is constant so $\alpha = \frac{d\omega}{dt} = 0$

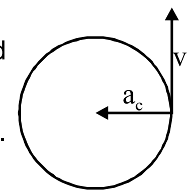
(vi) For non-uniform circular motion $a \neq 0$

Note : ❑ Relation between linear (tangential) acceleration and angular acceleration $\vec{a} = \vec{\alpha} \times \vec{r}$

❑ For uniform circular motion angular acceleration is zero, so tangential acceleration also is equal to zero.

❑ For non-uniform circular motion $a \neq 0$ (because $\alpha \neq 0$).

Centripetal Acceleration



(1) Acceleration acting on the object undergoing uniform circular motion is called centripetal acceleration.

(2) It always acts on the object along the radius towards the centre of the circular path.

(3) Magnitude of centripetal acceleration $a = \frac{v^2}{r} = \omega^2 r = 4\pi n^2 r = \frac{4\pi^2}{T^2} r$

(4) Direction of centripetal acceleration : It is always the same as that of $\Delta\vec{v}$. When Δt decreases, $\Delta\theta$ also decreases. Due to which $\Delta\vec{v}$ becomes more and more perpendicular to $\vec{v}$. When $\Delta t \to 0$, $\Delta\vec{v}$ becomes perpendicular to the velocity vector. As the velocity vector of the particle at an instant acts along the tangent to the circular path, therefore and hence the centripetal acceleration vector acts along the radius of the circular path at that point and is directed towards the centre of the circular path.

Centripetal Force

According to Newton's first law of motion, whenever a body moves along a straight line with the uniform velocity, no force is required to maintain that velocity. But when a body moves along a circular path with uniform speed, its direction changes continuously i.e. velocity keeps on changing on account of a change in direction. According to Newton's second law of motion, a change in the direction of motion of the body can take place only if some external force acts on the body.

Due to inertia, at every point of the circular path; the body tends to move along the tangent to the circular path at that point (in figure). Since every body has directional inertia, a velocity cannot change by itself and as such we have to apply a force. But this force should be such that it changes the direction of velocity and not its magnitude. This is possible only if the force acts perpendicular to the direction of velocity. Because the velocity is along the tangent, this force must be along the radius (because the radius of a circle at any point is perpendicular to the tangent at that point). Further, as this force is to move the body in a circular path, it must acts towards the centre. This centre-seeking force is called the centripetal force.

Hence, centripetal force is that force which is required to move a body in a circular path with uniform speed. The force acts on the body along the radius and towards centre.

(1) Formula for centripetal force : $F = \frac{mv^2}{r} = m\omega^2 r = m4\pi^2 n^2 r = \frac{m4\pi^2 r}{T^2}$

(2) Centripetal force in different situation

Situation	Centripetal Force
A particle tied to a string and whirled in a horizontal circle	Tension in the string
Vehicle taking a turn on a level road	Frictional force exerted by the road on the tyres
A vehicle on a speed breaker	Weight of the body or a component of weight
Revolution of earth around the sun	Gravitational force exerted by the sun
Electron revolving around the nucleus in an atom	Coulomb attraction exerted by the protons in the nucleus
A charged particle describing a circular path in a magnetic field	Magnetic force exerted by the agent that sets up the magnetic field

Centrifugal Force

It is an imaginary force due to incorporated effects of inertia. When a body is rotating in a circular path and the centripetal force vanishes, the body would leave the circular path. This imaginary force is given a name to explain the effects on inertia to the observer who is sharing the circular motion of the body. This inertial force is called centrifugal force. Thus centrifugal force is a fictitious force which has significance only in a rotating frame of reference.

SOLVED EXAMPLE

Example 22. An athlete completes one round of a circular track of radius 10 m in 40 sec. The distance covered by him in 2 min 20 sec is

(A) 70 m (B) 140 m

(C) 110 m (D) 220 m

Solution. **(D)**

$$\text{No. of revolution } (n) = \frac{\text{Total time of motion}}{\text{Time period}} = \frac{140\,\text{sec}}{40\,\text{sec}} = 3.5$$

Distance covered by an athlete in revolution = $(n(2\pi r) = 3.5 \times 2 \times \frac{22}{7} \times 10$ = 220 m.

Example 22. A flywheel rotates at a constant speed of 3000 rpm. The angle described by the shaft in radian in one second is

(A) 2π (B) 30π

(C) 100π (D) 3000π

Solution. **(C)**

Angular speed = 3000 rpm = 50 $r\pi s$ = 50 × 2π rad/sec = 100 π rad/sec

i.e. angle described by the shaft in one second is 100 π rad

Example 23. A scooter is going round a circular road of radius 100 m at a speed of 10 m/s. The angular speed of the scooter will be

(A) 0.01 rad/s (B) 0.1 rad/s

(C) 1 rad/s (D) 10 rad/s

Solution. **(B)**

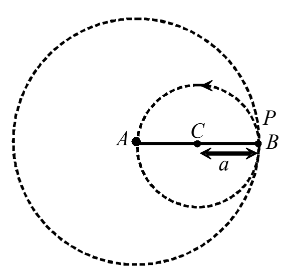
$$\omega = \frac{v}{r} = \frac{10}{100} = 0.1\,\text{rad/sec}$$

Example 24. A particle P is moving in a circle of radius 'a' with a uniform speed v. C is the centre of the circle and AB is a diameter. When passing through B the angular velocity of P about A and C are in the ratio

(A) 1 : 1 (B) 1 : 2

(C) 2 : 1 (D) 4 : 1

Solution. **(B)**



Angular velocity of P about A

$$\omega_A = \frac{v}{2a}$$

Angular velocity of P about C

$$\omega_C = \frac{v}{a}$$

$$\therefore \quad \frac{\omega_A}{\omega_C} = 1:2$$

Example 25. A body is whirled in a horizontal circle of radius 20 cm. It has angular velocity of 10 rad/s. What is its linear velocity at any point on circular path

(A) 10 m/s (B) 2 m/s

(C) 20 m/s (D) m/s

Solution. **(B)**

v = rω = 0.2 × 10 = 2 m/s

Example 26. What is the value of linear velocity, if $\vec{\omega} = 3\hat{i} - 4\hat{j} + \hat{k}$ and $\vec{r} = 5\hat{i} - 6\hat{j} + 6\hat{k}$

(A) $6\hat{i} + 2\hat{j} - 3\hat{k}$ (B) $18\hat{i} + 13\hat{j} - 2\hat{k}$

(C) $4\hat{i} - 13\hat{j} + 6\hat{k}$ (D) $6\hat{i} - 2\hat{j} + 8\hat{k}$

Solution. **(B)**

$$\vec{v} = \vec{\omega} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{J} & \hat{k} \\ 3 & -4 & 1 \\ 5 & -6 & 6 \end{vmatrix}$$

$$\vec{v} = (-24+6)i - (18-5)\hat{J} + (-18+20)\hat{k} = 18\hat{i} + 13\hat{J} - 2\hat{k}$$

Example 27. A body of mass 2 kg is revolved with a uniform speed in a horizontal circle of radius 2 meter. If the body completes 7 revolutions in 2 seconds, then find the centripetal acceleration and centripetal force acting upon the body $\pi = \frac{22}{7}$

Solution. The centripetal acceleration acting on the body in a horizontal circular path is given by

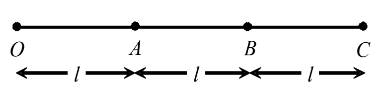
$a = \frac{v^2}{r} = r\omega^2$ where r is the radius of the path and ω is the angular

velocity of the body. Here r = 2 m and $\omega = \frac{2\pi n}{t} \Rightarrow 2 \times \frac{22}{7} \times \frac{7}{2} = 22\ \text{rads}^{-1}$

$\Rightarrow \quad a \Rightarrow r\omega^2 = 2 \times (22)^2 = 968\ \text{ms}^{-1}$.

Centripetal force = ma = 2 kg x 968 ms^{-1} = 1936 N

Example 28. Three identical particles are joined together by a thread as shown in figure. All the three particles are moving in a horizontal plane. If the velocity of the outermost particle is v_0, then the ratio of tensions in the three sections of the string is

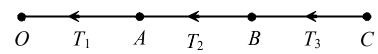


(A) 3 : 5 : 7 (B) 3 : 4 : 5

(C) 7 : 11 : 6 (D) 3 : 5 : 6

Solution. **(D)**

Let the angular speed of the thread is ω



For particle 'C' $\Rightarrow T_3 = m\omega^2 3l$

For particle 'B' $T_2 - T_3 = m\omega^2 3l \Rightarrow T_2 = m\omega^2 5^l$

For particle 'C' $T_1 - T_2 = m\omega^2 l \Rightarrow T_1 = m\omega^2 6l$

$\therefore \quad T_3 : T_2 : T_1 = 3 : 5 : 6$

Example 29. A proton of mass 1.6×10^{-27} kg goes round in a circular orbit of radius 0.10 m under a centripetal force of 4×10^{-13} N. then the frequency of revolution of the proton is about

(A) 0.08×10^8 cycles per sec (B) 4×10^8 cycles per sec

(C) 8×10^8 cycles per sec (D) 12×10^8 cycles per sec

Solution. **(A)**

$F = 4 \times 10^{-13}$ N ; $m = 1.6 \times 10^{-27}$ kg; r = 0.1 m

Centripetal force.

$F = m4\pi^2 n2r \;\therefore b = \sqrt{\dfrac{F}{4m^2 r}} = 8 \times 10^6$ cycles / sec = 0.08 × 108 cycle/sec

Example 30. A cord can bear a maximum force of 100 N without breaking. A body of mass 1 kg tied to one end of a cord of length 1 m is revolved in a horizontal plane. What is the maximum linear speed of the body so that the cord does not break

(A) 10 m/s (B) 20 m/s

(C) 25 m/s (D) 30 m/s

Solution. **(A)**

Tension in cord appears due to centrifugal force $T = \dfrac{mv^2}{r}$ and for critical condition this tension will be equal to breaking force (100 N)

$\therefore \dfrac{mv_{max}^2}{r} = 100 \Rightarrow v_{max}^2 = \dfrac{100 \times 1}{1} \Rightarrow v_{max} = 10 \text{ m/s}$

EXERCISE

29. A particle completes 1.5 revolutions in a circular path of radius 2 cm. The angular displacement of the particle will be – (in radian)

(A) 6π (B) 3π

(C) 2π (D) π

30. The length of the seconds hand of a watch is 10 mm. What is the change in the angular speed of the watch after 15 seconds

(A) Zero (B) $(10\pi/2)$ mms^{-1}

(C) $(20/\pi)$ mms^{-1} (D) $10\sqrt{2}$ mms^{-1}

31. A particle is moving along a circular path of radius 2 m and with uniform speed of 5 ms^{-1}. What will be the change in velocity when the particle completes half of the revolution

(A) Zero (B) 10 ms^{-1}

(C) $10\sqrt{2}\ ms^{-1}$ (D) $10/\sqrt{2}\ ms^{-1}$

32. The linear acceleration of a car is 10m/s^2. If the wheels of the car have a diameter of 1m, the angular acceleration of the wheels will be

(A) 10 rad/sec^2 (B) 20 rad/sec^2

(C) 1 rad/sec^2 (D) 2 rad/sec^2

33. The angular speed of a motor increases from 600 rpm to 1200 rpm in 10 s. What is the angular acceleration of the motor

(A) 600 rad sec^{-2} (B) 60π rad sec^{-2}

(C) 60 rad sec^{-2} (D) 2π rad sec^{-2}

34. A particle moves with a constant speed v along a circular path of radius r and completes the circle in time T. What is the acceleration of the particle

(A) mg (B) $\frac{2\pi v}{T}$

(C) $\frac{\pi r^2}{T}$ (D) $\frac{\pi v^2}{T}$

35. If the speed of revolution of a particle on the circumference of a circle and the speed gained in falling through a distance equal to half the radius are equal, then the centripetal acceleration will be

(A) $\frac{g}{2}$ (B) $\frac{g}{4}$

(C) $\frac{g}{3}$ (D) g

36. Two cars going round curve with speeds one at 90 km/h and other at 15 km/h. Each car experiences same acceleration. The radii of curves are in the ratio of

(A) 4 : 1 (B) 2 : 1

(C) 16 : 1 (D) 36 : 1

37. A ball of mass 0.1 kg is whirled in a horizontal circle of radius 1 m by means of a string at an initial speed of 10 r.p.m. Keeping the radius constant, the tension in the string is reduced to one quarter of its initial value. The new speed is

(A) 5 r.p.m. (B) 10 r.p.m.

(C) 20 r.p.m. (D) 14 r.p.m.

38. If mass speed and radius of rotation of a body moving in a circular path are all increased by 50%, the necessary force required to maintain the body moving in the circular path will have to be increased by

(A) 225% (B) 125%

(C) 150% (D) 100%